Conclusions

1) Theoretical and experimental investigations show that it can be assumed that cantilever vibration at constant frequency and amplitude of several cps and higher does not, for all practical purposes, influence the reading of a two-axis gyrocompass, meaning that, in a real model, its period T will not be less than 30 sec.

2) Experimental observations show that cantilever oscillation at a variable frequency and amplitude distorts the reading of a two-axis gyrocompass and, for all practical purposes, it becomes useless for determining the position of the meridian plane.

—Submitted December 7, 1960

Reference

[1] Il'in, P. A. and Sergeyev, M. A., "A terrestrial two-axis gyrocompass with spherical air bearings. Problems of the theory and calculation of gyroscopic instruments and precision instruments", LITMO (Leningrad Institute of Precision Mechanics and Optics), no. 36 (1958).

Reviewer's Comment

The reviewer considers Fig. 1 the primary asset of this paper, since it tells us quite a bit about the state of the art of Russian gyro technology.

In addition, it is of interest to follow the mathematical derivations, which appear to be correct. The reviewer arrives at the conclusion that the experimental observations, which are in themselves revealing, are not adequately explained by the theoretical analysis.

—C. L. Emmerich
Stabilization and Navigation Dept.
Norden Division
United Aircraft Corporation

JULY 1963 AIAA JOURNAL VOL. 1, NO. 7

Method of Superposition under Conditions of Elasticity and Destructive Stress

V. D. Kliushnikov

THIS note is intended to show that, given the ordinary assumptions about the connection between stresses and deformations in a system consisting of a plane and a direction, the method of superposition, analogous to the Batdorf-Budiansky method in the theory of slip,[1] can be used to obtain the ordinary relations of elasticity. This result, which confirms the validity of the theory of slip, is also of interest in that it evidently brings out a new aspect of the problem of the correlation between the theoretical and observed stresses of brittle fracture.

Let us take a given deformation ϵ_{ij} in the system of axes x, y, z. Then an element with normal n will experience the normal deformation

$$\epsilon_{nn} = l_{in}l_{jn}\epsilon_{ij} \tag{1}$$

and in the direction m a shear deformation

$$\epsilon_{nm} = l_{in}l_{jm}\epsilon_{ij} \tag{2}$$

Here l_{in}, l_{im} are the direction cosines in the system of axes x, y, z of the normal n and the direction m, respectively. As a result of this, the element with normal n is subjected to a normal stress σ_{nn} and a tangential stress σ_{nm} in the direction m. In the axes x, y, z the stresses σ_{nn} and σ_{nm} produce stresses

$$\sigma_{ij}^{\circ} = \sigma_{nn}l_{in}l_{jn} + \sigma_{nm}(l_{in}l_{jm} + l_{im}l_{jn}) \tag{3}$$

Assuming that the total stress σ_{ij} in the axes x, y, z is the average of these unit stresses with respect to all possible surface elements and directions within them (this assumption coincides with the procedure in the theory of slip except insofar as stresses are substituted for deformations), we get

$$\sigma_{ij} = \frac{1}{Q_1}\int_{Q_1} \sigma_{nn}l_{in}l_{jn}dQ_1 + \frac{1}{Q_2}\int_{Q_2} \sigma_{nm}(l_{in}l_{im} + l_{jm}l_{jn})dQ_2 \tag{4}$$

$$dQ_1 = d\Omega \qquad dQ_2 = d\Omega d\beta \tag{5}$$

Here Ω is the solid angle, and β is the angle formed by the direction m at the element with normal n and a certain fixed direction.

The direction cosines l_{in} and l_{im} can easily be expressed in terms of longitude α and latitude φ on a unit sphere and the angle β, as in the theory of slip:[1]

$$\begin{aligned} l_{xn} &= \sin\alpha \cos\varphi \\ l_{yn} &= \cos\alpha \cos\varphi \\ l_{zn} &= \sin\varphi \\ l_{xm} &= \cos\alpha \sin\beta - \sin\alpha \cos\beta \sin\varphi \\ l_{ym} &= -\sin\alpha \sin\beta - \cos\alpha \cos\beta \sin\varphi \\ l_{zm} &= \cos\beta \cos\varphi \end{aligned} \tag{6}$$

where

$$d\Omega = \cos\varphi d\varphi d\alpha \tag{7}$$

In investigating the validity of this procedure for obtaining the connection between stresses and deformations in the elastic case, it is natural to assume that in a system consisting of a plane and a direction the stresses and deformations are connected by the relations

$$\sigma_{nn} = a\epsilon_{nn} \qquad \sigma_{nm} = b\epsilon_{nm} \qquad (a,b = \text{const}) \tag{8}$$

As distinct from the case of plastic deformations, in the case in question integration must be carried out with respect

Translated from Zhurnal Prikladnoi Mekhaniki i Technicheskoi Fiziki (Journal of Applied Mechanics and Technical Physics), no. 3, 103–104 (1961). Translated by Faraday Translations, New York.

[1] Batdorf, S. B. and Budiansky, B. A., "Mathematical theory of plasticity based on the concept of slip," NACA TN 1871 (April 1949).

to all the planes and all the directions in them, i.e., within the limits

$$0 \leqslant \alpha \leqslant 2\pi \qquad -\pi/2 \leqslant \varphi \leqslant \pi/2$$
$$-\pi/2 \leqslant \beta \leqslant \pi/2 \tag{9}$$

where

$$Q_1 = 1/4\pi \qquad Q_2 = 1/4\pi^2 \tag{10}$$

On the basis of this and relations (8), Eqs. (4) assume the form

$$\sigma_{ij} = \frac{a}{4\pi}\int_0^{2\pi} d\alpha \int_{-\pi/2}^{\pi/2} d\varphi[\epsilon_{nn}l_{in}l_{jn}] + \frac{b}{4\pi^2}\int_0^{2\pi} d\alpha \times \int_{-\pi/2}^{\pi/2} d\varphi \int_{-\pi/2}^{\pi/2} d\beta[\epsilon_{nm}(l_{in}l_{jm} + l_{im}l_{jn})] \tag{11}$$

Taking into account Eqs. (6), after integration we get

$$\sigma_{ij} = \lambda\theta\delta_{ij} + 2\mu\epsilon_{ij} \tag{12}$$

that is, the ordinary generalization of Hooke's law, where

$$\lambda = (a + b)/15 \qquad \mu = (2a + 3b)/30 \tag{13}$$

On the basis of known equations connecting Lamé's constants with Young's modulus E and Poisson's ratio ν we get

$$E = \frac{a}{3}\left[\frac{2a + 3b}{4a + b}\right] \qquad \nu = \frac{a - b}{4a + b} \tag{14}$$

Whence it follows that

$$a = 3E/(1 - 2\nu) = 3K \tag{15}$$

Here K is the modulus of bulk compression.

Suppose that the yield stress, measured in a uniaxial tension test, is σ_H, whereas the theoretical stress, calculated from conditions of rupture of the atomic planes, is σ_T. The maximum tearing stresses are experienced by atomic planes perpendicular to the direction in which the tension is applied. By virtue of relations (6)

$$\sigma_{nn} = \sigma_T = a\epsilon_{nn} = a\epsilon \tag{16}$$

where ϵ is the axial deformation. On the other hand, $\epsilon = \sigma_0/E;$ but then on the basis of Eq. (16) we have

$$\sigma_H = 3(1 - 2\nu)\sigma_T \tag{17}$$

From this it follows that, as ν approaches $\frac{1}{2}$ (incompressible body), the observed rupture stress may be much less than the theoretical stress. Although (in view of the asymmetry of the method with respect to stresses and deformations) this cannot be regarded as a new explanation of the sharp difference between the theoretical and observed rupture stresses, a more critical approach is necessary to existing calculations of theoretical stress with regard to interactions not coinciding with the direction of rupture.

—*Submitted May 18, 1961*

JULY 1963 AIAA JOURNAL VOL. 1, NO. 7

Cycles about a Singular Point of Nodal Type

A. P. Vorob'ev
Institute for Mathematics and Computational Technology,
Academy of Sciences, White Russian Soviet Republic

***In a differential equation system in the plane**

$$dx/dt = P(x, y) \qquad dy/dt = Q(x, y)$$

where P and Q are polynomials, with $P(0,0) = Q(0,0) = 0$, there may exist closed trajectories enclosing the singular point at the origin. If the degrees of P and Q are at most two, and the origin is a nodal point, it is shown that there can be no closed trajectory enclosing the origin. Sufficient conditions for a nodal point to be acyclic (i.e., not to be enclosed by a closed trajectory) are developed.

It is known that for a system of differential equations of the form

$$\frac{dx}{dt} = \sum_{i+j=1}^{n} a_{ij}x^iy^j$$
$$\frac{dy}{dt} = \sum_{i+j=1}^{n} b_{ij}x^iy^j \tag{1}$$

(a_{ij},b_{ij} constant) that when $n = 1$ (a linear system) a singular point of nodal type cannot be enclosed by a closed trajectory. This is possible for $n \geqq 3$. For example, in the system

$$dx/dt = -y + ax(x^2 + y^2 - 1)$$
$$dy/dt = x + by(x^2 + y^2 - 1)$$

with $ab \geqq -1$, $(a - b)^2 \geqq 4$, the singular point at (0,0) is a node, and it is enclosed by the closed trajectory $x^2 + y^2 = 1$.

In the present note we shall formulate sufficient conditions that a singular point be acyclic,† and shall show that in the system (1) with $n = 2$ a singular point of nodal type cannot be enclosed by a closed solution.

Consider the system of differential equations

$$d\rho/dt = F(\rho,\theta) \qquad d\theta/dt = \Phi(\rho,\theta) \tag{2}$$

(where ρ and θ are polar coordinates and $F(0,\theta) \equiv 0$), for which conditions for existence and uniqueness of solutions are fulfilled throughout the entire plane.

Translated from Doklady Akademii Nauk BSSR (Proceedings of the Academy of Sciences, White Russian Soviet Republic) 4, No. 9, 369–371 (1960). Translated by Warren S. Loud, Department of Mathematics, University of Minnesota, Minneapolis, Minn. Reviewed for publication by Lawrence Markus, Department of Mathematics, University of Minnesota, Minneapolis, Minn.

* This abstract was prepared by Professor Warren S. Loud.

† By an acyclic singular point we shall mean a singular point not enclosed by a closed trajectory.